Analyzing Performance of Joint SVR Interpolation for LTE System with 64-QAM Modulation under 500 Km/h Mobile Velocity

Anis CHARRADA, EPT,
Carthage University
Abstract

Context:
- Support Vector Machine Regression
- LTE Communication System

Problematic:
- Estimate the coefficients of a frequency selective time varying multi-path fading under very high mobility conditions

Proposed solution:
- Complex SVR algorithm based on kernel functions and using a robust cost function

Results:
- Best performances are obtained by Complex SVR
Outline

1. OFDM System Model
2. Complex SVR approach
3. Simulation Results
4. Conclusion
OFDM System Model

**Time domain transmitted signal**

\[ x(n) = \text{IDFT}_N\{X(k)\} = \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}}, \quad n = 0, \ldots, N - 1. \]

- \( x(n) \): time domain transmitted signal,
- \( N \): numero de subcarriers,
- \( X(k) \): complex QAM symbols.

**Time domain received signal**

\[ y(n) = \sum_{k=0}^{N-1} X(k) H(k) e^{j\frac{2\pi kn}{N}} + w_g(n), \quad n = 0, \ldots, N - 1. \]

- \( y(n) \): time domain received signal,
- \( H(k) = \text{DFT}_N\{h(n)\} \): channel's frequency response at the \( k \)th subcarrier,
- \( w_g(n) \): complex white Gaussian noise \( N(0, \sigma_{w_g}^2) \).
**OFDM System Model**

### Time domain received signal

\[ y(n) = y^P(n) + y^D(n) + w_g(n) \]

\[ y(n) = \sum_{k \in \Omega_P} X^P(k) H(k) e^{j \frac{2\pi}{N} kn} + \sum_{k \in \Omega_P} X^D(k) H(k) e^{j \frac{2\pi}{N} kn} + w_g(n) \]

- \(X^P(k)\): complex pilot symbol transmitted at the \(k^{th}\) subcarrier,
- \(X^D(k)\): complex data symbol transmitted at the \(k^{th}\) subcarrier.
- \(w_g\): Additive White Gaussian Noise.

### Frequency domain received signal

\[ Y(k) = DFT_N\{y(n)\} = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j \frac{2\pi}{N} kn}, \quad k = 0, \ldots, N - 1. \]

\[ Y(k) = X(k)H(k) + W_G \]

\[ Y = XH + W_G \quad \text{Matrix notation} \]

\[ Y = [Y(0), \ldots, Y(N - 1)]^T \]

\[ X = \text{diag}(X(0), X(1), \ldots, X(N - 1)) \]

\[ H = [H(0), \ldots, H(N - 1)]^T \]

\[ W_G = [W_G(0), \ldots, W_G(N - 1)]^T. \]
The transmitting pilot symbols:
\[ X^P = \text{diag}(X(i,m\Delta p)), \quad m = 0, \cdots, N_P - 1. \]
\( \Delta p \): pilot intervall in frequency domain.
\( N_P \): Number of pilot symbols.

**Training phase**
\[ \hat{H}^P = X^{P^{-1}} Y^P, \]
\[ Y^P = Y(i,m\Delta p): \text{received pilot symbol at pilot position } m\Delta p \]
\[ \hat{H}^P = \hat{H}(i,m\Delta p): \text{frequency response at pilot position } m\Delta p. \]

**Estimation phase**
\[ \hat{H}(i,k) = f(\hat{H}^P(i,m\Delta p)) \]
\( f(\cdot): \text{SVR interpolating function for all data subcarriers, } \]
\[ k = 0, \cdots, N - 1. \]
Complex SVR Approach

\[ \minimize w \]

\[ \frac{1}{2} ||w||^2 + \sum_{m=1}^{M} (\mathcal{L}^\varepsilon(\Re(e_m), \varepsilon, \gamma, C) + \mathcal{L}^\varepsilon(\Im(e_m), \varepsilon, \gamma, C)) \]

\( \varepsilon \) - insensitivity zone and losses
\[ \mathcal{L}^\varepsilon(e_m, \varepsilon, \gamma, C) = \begin{cases} 
0, & |e_m| \leq \varepsilon \\
\frac{1}{2\gamma}(|e_m| - \varepsilon)^2, & \varepsilon \leq |e_m| \leq \gamma C + \varepsilon \\
C(|e_m| - \varepsilon) - \frac{1}{2} \gamma C^2, & |e_m| \geq \gamma C + \varepsilon 
\end{cases} \]

**\varepsilon-Huber Loss Function**

- **\(I_0\): \varepsilon\)-insensitivity zone
- **\(I_1\): zone with quadratic cost
- **\(I_2\): zone with linear cost**
## Complex SVR Approach

### SVR Estimator Formulation

\[
\hat{H}(m\Delta p) = w^H \phi(m\Delta p) + b + e_m, \quad m = 0, \ldots, N_p - 1.
\]

### Primal problem

**minimize**  
\[
\frac{1}{2} \|w\|^2 + \frac{1}{2\gamma} \sum_{m \in I_1} (\xi_m + \xi_m^*)^2 + C \sum_{m \in I_2} (\xi_m + \xi_m^*) \\
+ \frac{1}{2\gamma} \sum_{m \in I_3} (\zeta_m + \zeta_m^*)^2 + C \sum_{m \in I_4} (\zeta_m + \zeta_m^*) - \frac{1}{2} \sum_{m \in I_2,I_4} \gamma C^2
\]

**s. t.**  
\[
\Re(\hat{H}(m\Delta p) - w^H \phi(m\Delta p) - b) \leq \varepsilon + \xi_m \\
\Im(\hat{H}(m\Delta p) - w^H \phi(m\Delta p) - b) \leq \varepsilon + \zeta_m \\
\Re(-\hat{H}(m\Delta p) + w^H \phi(m\Delta p) + b) \leq \varepsilon + \xi_m^* \\
\Im(-\hat{H}(m\Delta p) + w^H \phi(m\Delta p) + b) \leq \varepsilon + \zeta_m^* \\
\xi_m, \xi_m^*, \zeta_m, \zeta_m^* \geq 0.
\]
After derivation of the Lagrangian with respect to primal variables, we obtain:

\[ w = \sum_{m=0}^{N_P-1} \psi_m \phi(m\Delta p) \]

\[ \psi_m = (\alpha_{\Re,m} - \alpha_{\Re,m}^*) + j(\alpha_{\Im,m} - \alpha_{\Im,m}^*) \]

\[ \alpha_{\Re,m}, \alpha_{\Re,m}^*, \alpha_{\Im,m}, \alpha_{\Im,m}^* \geq 0. \]

\[ \hat{H}(k) = \sum_{m=0}^{N_P-1} \psi_m K(m\Delta p, k) + b \]

\[ k = 1, \ldots, N. \]

\[ b : \text{is the biasterm.} \]

\[ G(u, v) = <\phi(u\Delta p) \cdot \phi(v\Delta p)> = K(u\Delta p, v\Delta p) \]
Simulation results

LTE Downlink system

Power delay profile for LTE – 3GPP Extended Vehicular A (EVA) channel

<table>
<thead>
<tr>
<th>Excess tap delay (ns)</th>
<th>Relative power of taps (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>30</td>
<td>-1.5</td>
</tr>
<tr>
<td>150</td>
<td>-1.4</td>
</tr>
<tr>
<td>310</td>
<td>-3.6</td>
</tr>
<tr>
<td>370</td>
<td>-0.6</td>
</tr>
<tr>
<td>710</td>
<td>-9.1</td>
</tr>
<tr>
<td>1090</td>
<td>-7.0</td>
</tr>
<tr>
<td>1730</td>
<td>-12.0</td>
</tr>
<tr>
<td>2510</td>
<td>-16.9</td>
</tr>
</tbody>
</table>

Variations in time and in frequency (mobile speed = 500Km/h)
### Simulation results

**LTE Downlink system**

\[
SNR_{dB} = 10 \log_{10} \left( \frac{E \left\{ |y(n) - w_g(n) - i(n)|^2 \right\}}{\sigma_{w_g}^2} \right)
\]

**Simulation system**

- 1400 OFDM symbols = 10 radio frame LTE.
- Radio frame LTE duration = 10 ms.
- 01 Radio frame LTE = 10 subframes.
- 01 subframe = 02 slots.
- 01 slot duration = 0.5 ms.

---

### LTE – 3GPP simulation parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFDM System</td>
<td>LTE / Downlink</td>
</tr>
<tr>
<td>Constellation</td>
<td>64-QAM</td>
</tr>
<tr>
<td>Mobile Speed (Km/h)</td>
<td>500</td>
</tr>
<tr>
<td>(T_s(\mu s))</td>
<td>72</td>
</tr>
<tr>
<td>(f_c(\text{GHz}))</td>
<td>2.15</td>
</tr>
<tr>
<td>(\delta f(\text{KHz}))</td>
<td>15</td>
</tr>
<tr>
<td>(B(\text{MHz}))</td>
<td>5</td>
</tr>
<tr>
<td>Size of DFT/IDFT</td>
<td>512</td>
</tr>
<tr>
<td>Number of paths</td>
<td>9</td>
</tr>
</tbody>
</table>

**SVR parameters**

\(C = 100, \gamma = 10^{-5}, \varepsilon = .001\)
Simulation results

BER vs. SNR

BER as a function of SNR
Simulation results

MSE vs. SNR

MSE as a function of SNR

- LS Estimation
- Feedback Estimation
- Neural Network Estimation
- SVR Estimation
Conclusion

Complex SVR approach has been developed and applied to LTE Downlink System under high mobility conditions (500 Km/h) using 6-QAM modulation scheme with real scenarios according to 3GPP specifications.

Future Work

Apply the complex SVR in:

- LTE-Advanced and 5G systems.
References


Acknowledgments

Acknowledgments to all members of SERCOM_LAB Group.
Thank You for Your Attention