Some Geolocation Determination Techniques

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COMNET, Hamamet, Tunisia

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Outline for Part 1: Generalities

- A few words on history of wireless **geolocation** or **position determination**
- Discussion of a few application scenarios
- Major systems
- Major localization techniques
- Some terminology
- Formulation of RSS based localization problems
- Some results from our work
- Questions/Answers and discussion

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Major Existing Systems

- Global Positioning System (GPS)
- Positioning using existing cellular system
- Wi-Fi based positioning (indoor, outdoor)
- RFID based systems
- All sorts of specialized systems

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A few application scenarios

- Finding your way (outdoor) GPS
- Tracking equipment
  - Hospitals
  - Airports
  - Car dealer lots
  - Find my phone
- Tracking people
  - Criminals
  - Fire fighters
  - Elderly at home
  - Children outside
  - Everybody for advertisements

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A few more application scenarios

- Tracking animals in the wild (outdoor)
- Helping blind people move around
- Cognitive Radio (determining spectral holes)

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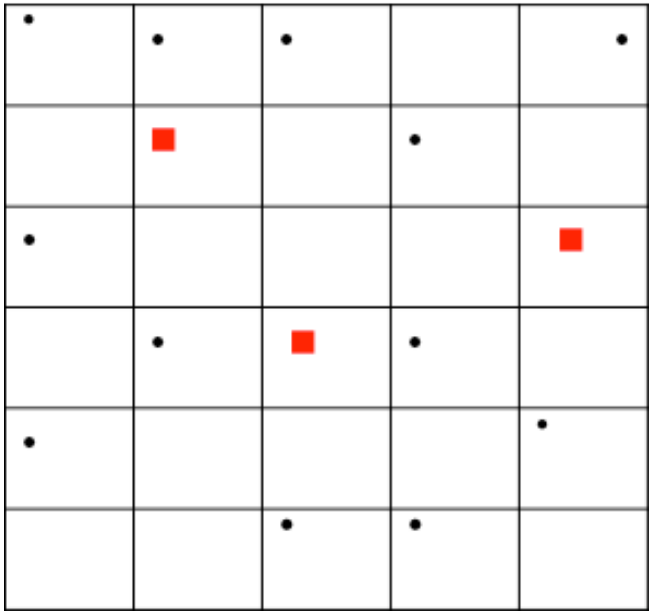
Problem Definition

- Cognitive radio networks consist of **a number of secondary users** deployed at known but arbitrary locations in a given geographical area.
- **One or more primary users are** active and each sensor measures the total received power due to all of the primary transmitters.



■ Emitter/tx (or Primary user (PU))

• Sensor/rx (or Secondary user (SU))

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The problems to solve

(a) Estimate the location of primary emitter(s)

(b) Estimate the transmit powers of the emitters

(c) Estimate the field strength values at different points by using interpolation.

Performance Metrics

- Accuracy (in meters)
- Speed (the time it requires to obtain the estimate)
- Cost
- Size
- Battery requirement

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Some terminology

- Indoor vs Outdoor localization
- Node: Anchor vs sensor
- Fingerprinting
- Radio Environment Map (REM)
- Cartography

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Techniques Utilized

- Time of arrival (TOA)
  - Requires accurate synchronization
  - Works well if direct line of sight is available
- Time Difference of Arrival (TDOA)
- Angle of Arrival (AOA)
  - Requires multiple antennas
- Received Signal Strength (RSS)
  - Simple but not very accurate without anchors or many sensors

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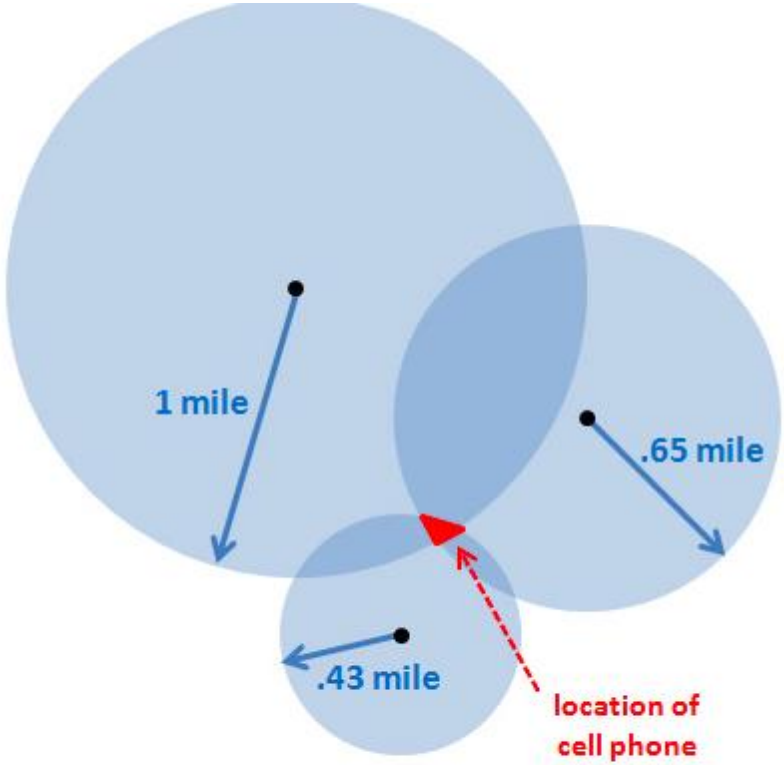
Triangulation



Triangulation - cell phone detected within a certain radius of each of 3 cell towers – the area where each cell tower overlaps the phone is where it is pinpointed.

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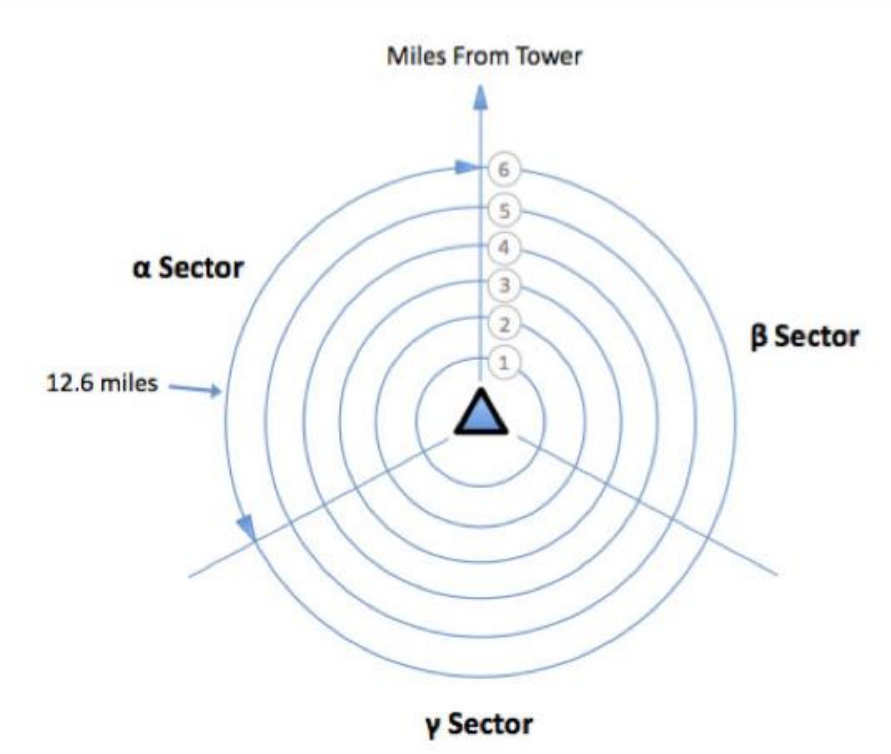
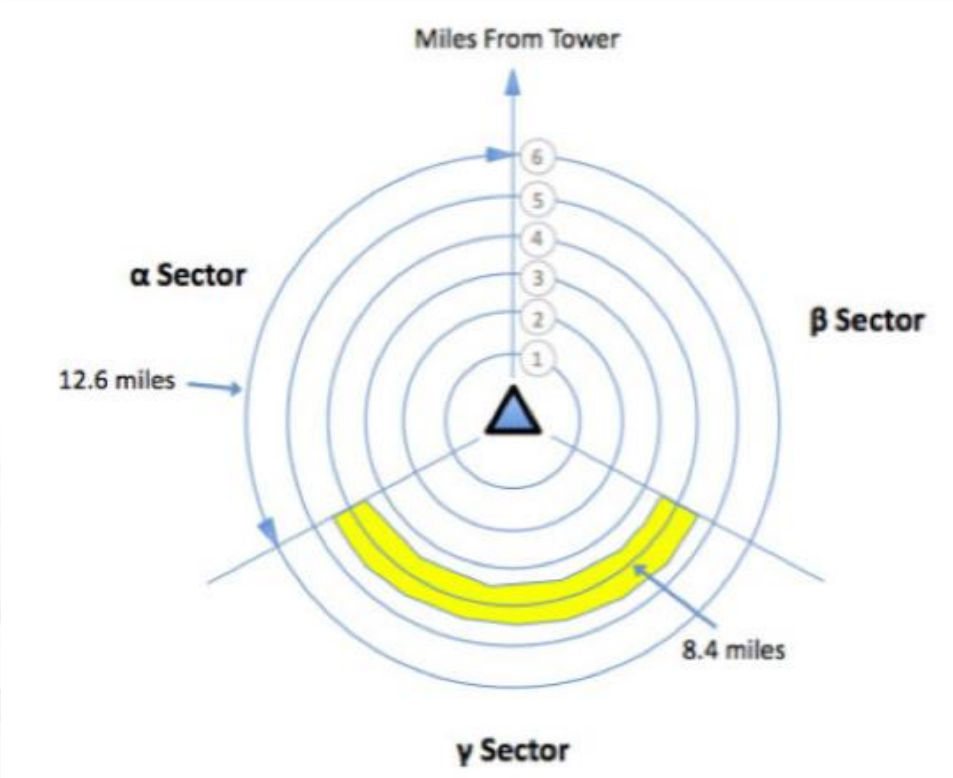
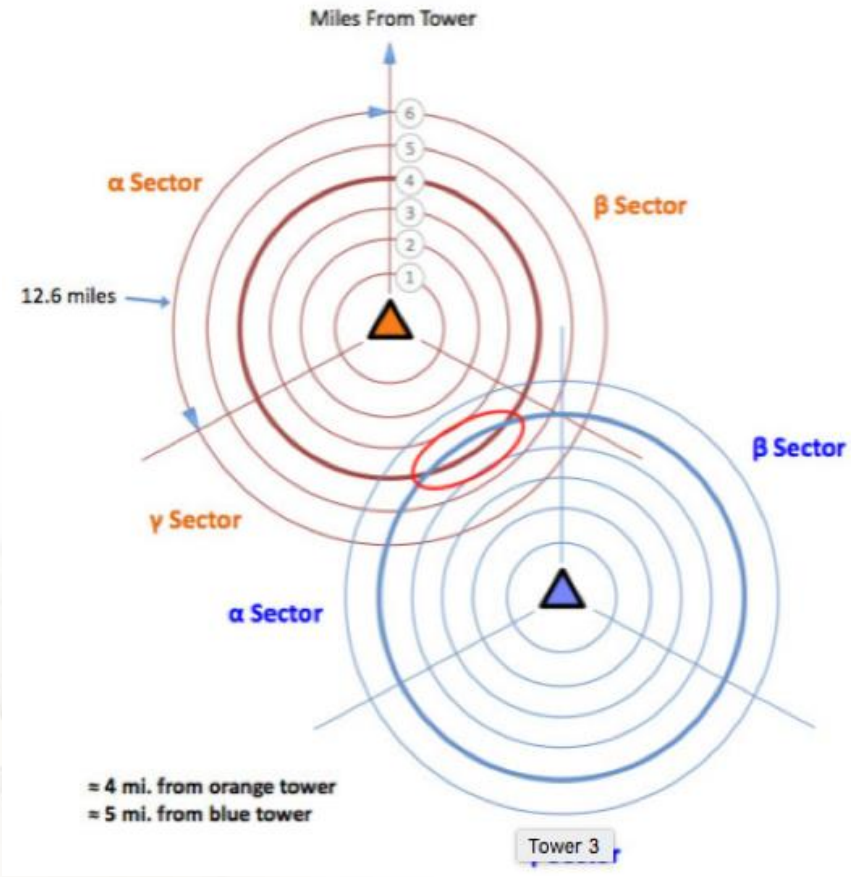
Localization using cell towers



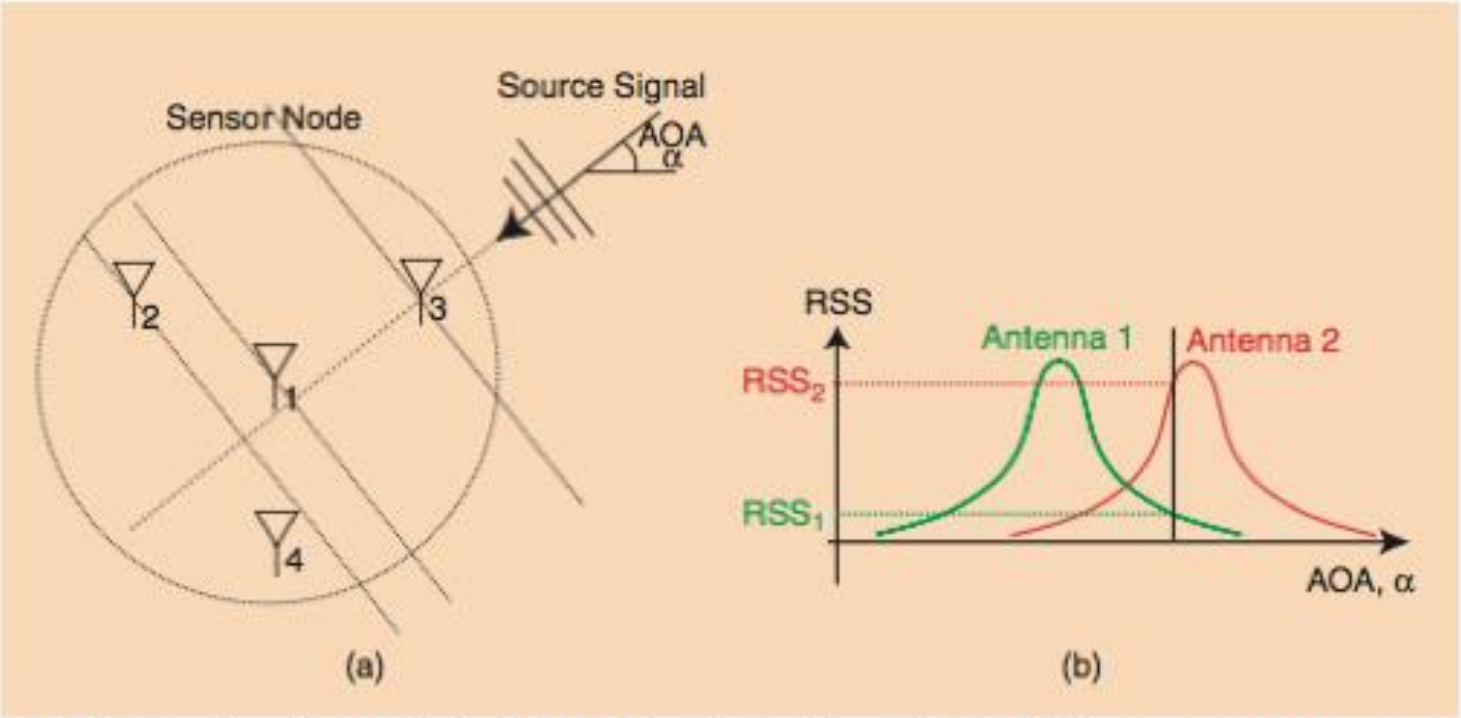
Localization using cell towers (ctd)



Localization using cell towers (ctd)



Angle of Arrival



[FIG3] AOA estimation methods. (a) AOA is estimated from the TOA differences among sensor elements embedded in the node; a four-element Y-shaped array is shown. (b) AOA can also be estimated from the RSS ratio RSS_1/RSS_2 between directional antennas.

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Time of Arrival (TOA)

In TOA based technique, the distance information is extracted from propagation delay between a TX and a RX.

TOA can be further classified into one-way ranging and two-way-ranging.

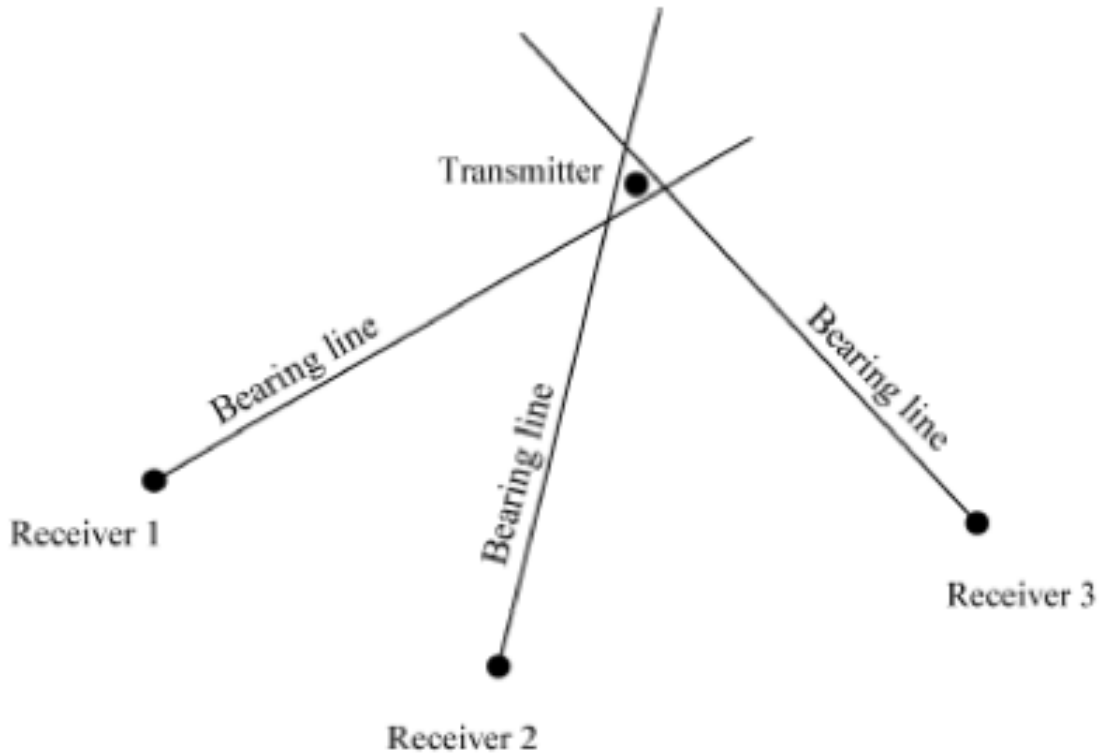
- One-way requires perfect synchronization between TX and RX.
- Two-way is more common in cellular networks since the receiving nodes are typically synchronized to base stations.

Time Difference of Arrival

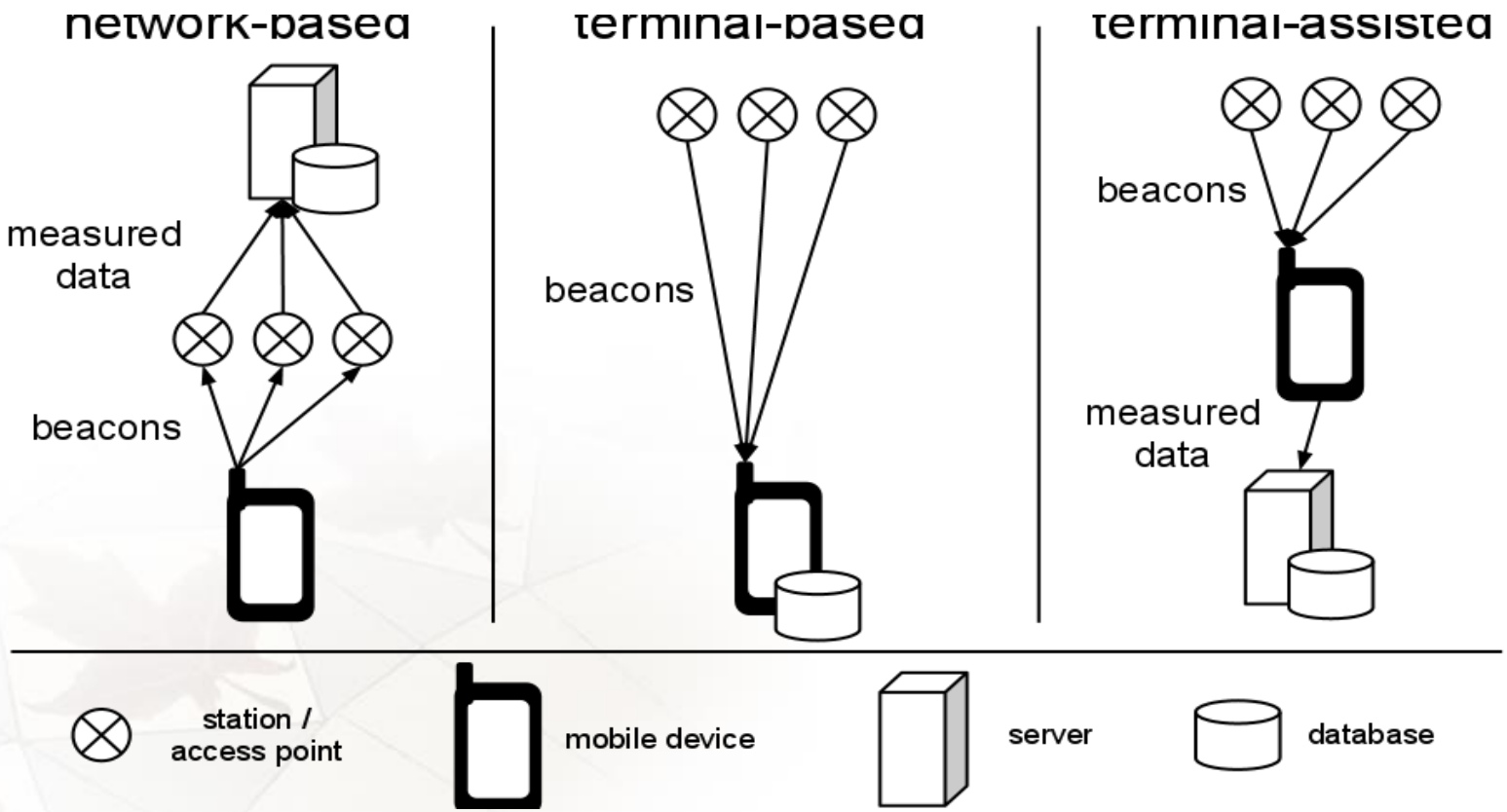
In TDOA the differences between TOAs in several RXs are used to reconstruct a TX's position.

- This could either be based on the difference in the times at which a single signal from a single node arrives at three or more nodes, or based on the difference in the times at which multiple signals from a single node arrive at another node.

Angle of Arrival



System Configurations



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Motivations

- Cognitive radios provide mechanisms that support dynamic spectrum access paradigms to achieve efficient utilization of the spectrum.
- A radio environment map is a tool that supports the operation of cognitive radio networks.
- Generation of an interference map requires measuring or estimating (via interpolation) the field strength at every point of interest.
- An interference map can determine where and when a secondary user can transmit.
- Primary emitters' locations and the transmit powers can be used to set a protected region around each primary emitter, preventing harmful interference to primary users.

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Outline for Part 2: some of our work

- Determining (known power) transmitter's location when distances to it are known at 3 points (trilateration)
- Determining the unknown power transmitter's location when received signal strength at 3 points are known
- Effects of path loss exponent and log-normal shadowing
- Interpolation techniques
- Least squares estimation
- Grid search estimation
- General mathematical model
- Using Metropolis-Hastings
- Closing remarks

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Determining Tx Location using RSS

We measure the rxd signal strength.

We can determine the distance to the tx if we know the propagation conditions and the transmit power.

Example: Free-space transmission. Signal attenuates proportional to the squared distance.

$P_T = \text{Transmit power}$
$P_R = \text{Received power}$
$k_1 = \text{a constant for tx and rx antenna gains etc.}$

$d = \text{distance between tx and rx}$

$$P_R = k_1 \frac{P_T}{d^2} \quad \rightarrow \quad d = \sqrt{\frac{k_1 P_T}{P_R}}$$

Case 1: Single Transmitter of known power

Basic Assumptions

1. The transmitter power is known
2. The path loss exponent is known

- Power received is measured at sensors 1, 2 and 3.
- We can determine the distances between the sensors and the transmitter as d_1, d_2 and d_3 (see next page)
- Using trilateration (often called triangulation, somehow incorrectly), the location of the transmitter can be determined (see 2 slides later)

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Received Power and determining the distance

P_T = Transmit power

P_R = Received power

β = a constant for accounting for tx and rx antenna gains etc.

$\beta = \beta_0 d_0$

β_0 = path loss at reference distance d_0.

Let d = distance between tx and rx

and α = path loss exponent

$P_R = \beta_0 P_T (\frac{d_0}{d})^\alpha$

If no other info, take $d_0 = 1\text{m}$, and β_0 as free space path loss.

$P_R = \beta_0 \frac{P_T}{d^\alpha}$ or $P_R(\text{dBm}) = \beta_0(\text{dB}) + P_T(\text{dBm}) - 10\alpha \log d$

$d = (\frac{\beta_0 P_T}{P_R})^{1/\alpha}$

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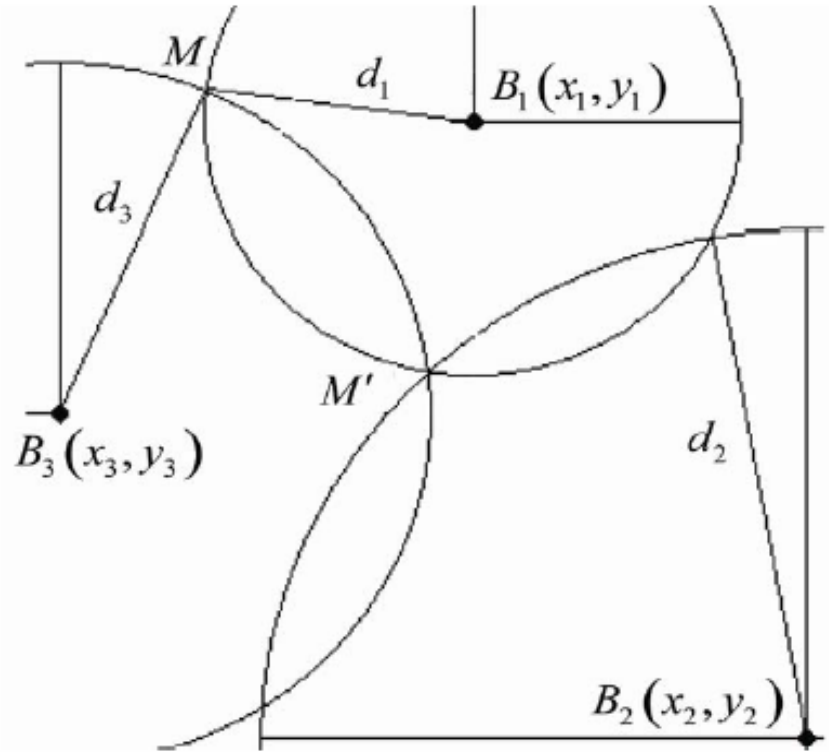
Trilateration

Given the distances from 3 points, the location of the tx can be uniquely determined.



Question

- For the case of known transmit power, we assumed path loss exponent $\alpha = 2$. If α is not 2, do the circles used in the trilateration still represent the locus of points of interest?

What if we don’t know the transmit power?

- Can we still determine the location of the transmitter based on the 3 received powers?

Case 2: Single Transmitter of unknown power

Basic Assumptions

1. The transmitter power is unknown
2. The path loss exponent is known

- Power received is measured at sensors 1, 2 and 3.
- **How do we determine the location of the transmitter?**

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Case 2 unknown tx power (continued)

- We cannot determine the distances d_1, d_2 etc but we can determine the locus of points that lie on a conic section (e.g. a circle) which represents the distances for a constant power ratio.

Consider tx power of P rx'd at 2 sensors as P_1 and P_2.

$$P_1 = \beta_0 \frac{P}{d_1^\alpha}$$

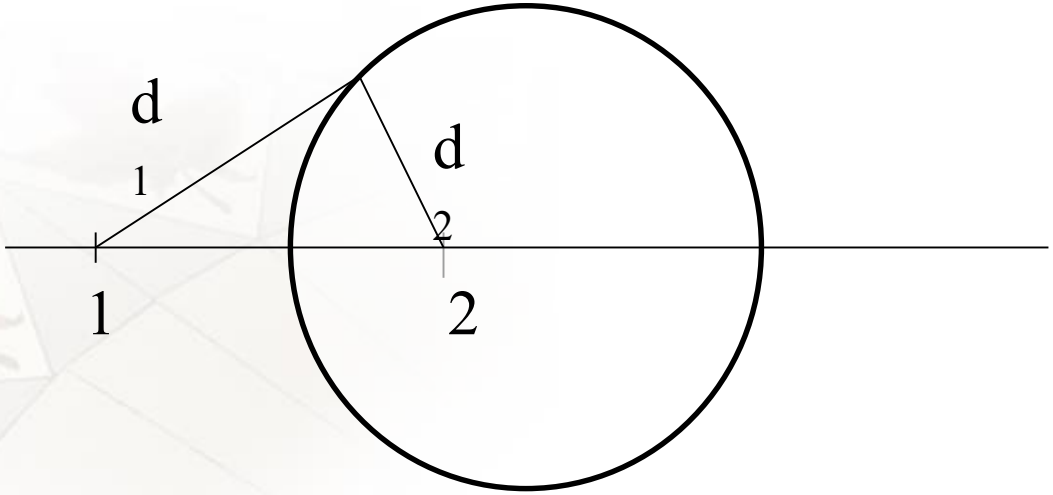
and

$$P_2 = \beta_0 \frac{P}{d_2^\alpha}$$

$$\frac{P_1}{P_2} = \left(\frac{d_2}{d_1}\right)^\alpha$$

Locus of points with a constant power ratio

1 and 2 represent the receiver locations receiving power P_1 and P_2. Let $P_2=2P_1$.Then the circle represents the locus of points satisfying this power ratio, given path loss exponent of 2.



Determining location using power ratios

- From the previous slide, we can observe that the points where a transmitter with unknown power can be located is not unique under certain conditions. For instance if the transmitter was on the same line with 1 and 2, the same power ratio would point either to a point between the two receivers, or to a point to the right side of receiver 2.
- If a third receiver is arbitrarily placed, except in some special cases, the power ratio would point to the correct location. So we could draw the second circle (or the corresponding shape, dependent on path loss exponent α)

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Unique Solution

Theorem: There is a unique solution equal to actual location and transmit power of transmitter, except when the monitors are placed on an arc of a circle, or a straight line that does not pass through the actual transmitter location

ZAFER *et al.*: TRANSMIT POWER ESTIMATION USING SPATIALLY DIVERSE MEASUREMENTS UNDER WIRELESS FADING, IEEE Trans. Networking, Aug. 2010

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Geometry for non-unique solution

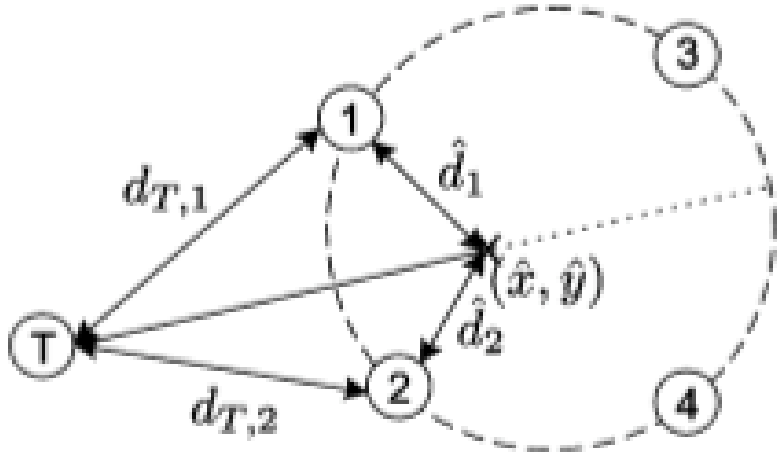

Fig. 1. Schematic diagram showing the placement of monitors on a circular ar that gives two solutions—transmitter location and position marked X.

Not unique solution

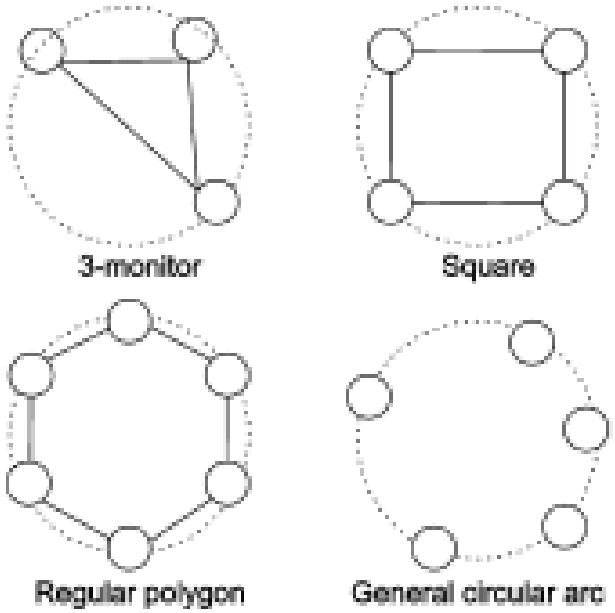

Fig. 2. Illustration of various monitor topologies that do not give a unique solution.

Recap

- If transmit power (and path loss exponent α) is known, using the received power and trilateration, location of the emitter can be determined.
- If transmit power is unknown, using the received power ratios and trilateration, location of the emitter can be determined.

Question

- For the transmit power unknown case, we assumed path loss exponent $\alpha = 2$. If α is not 2, do the circles used in the trilateration still represent the locus of points of interest?

Let us complicate the picture

- Transmit power is unknown
- Path loss exponent α is not known, or varies
- There is shadowing (e.g. log-normal) with a spread of σ^2 dB
- There are multiple transmitters
- The number of transmitters is unknown

Then what are the problems and how do we proceed?

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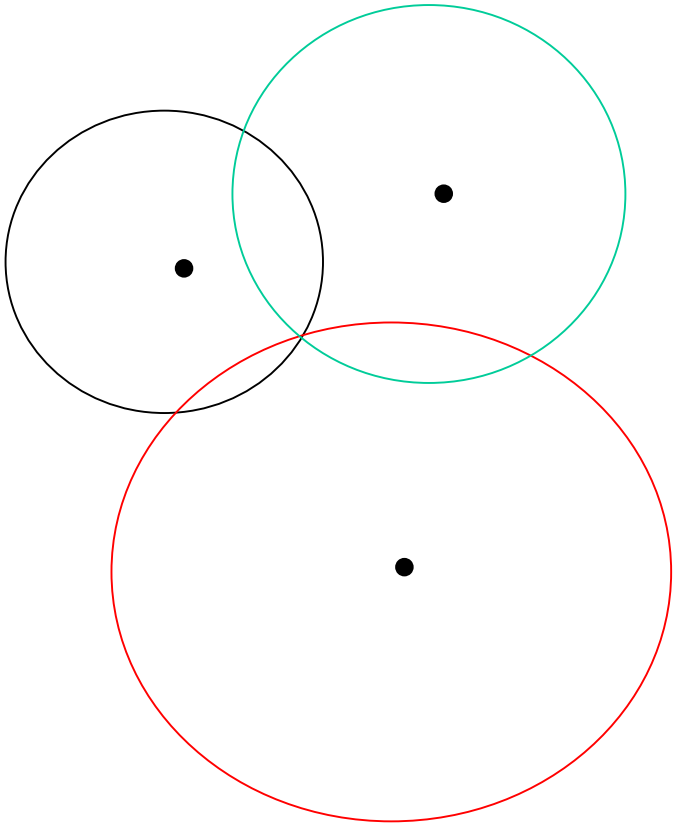
What are the possible problems with trilateration?

1. We need to know the transmitter power and the channel (i.e. path loss exponent) to determine the distances.

2. The sizes of the circles can vary considerably as a function of path loss exponent or shadowing spread.



Effect of path loss exponent α

Circles represent the range (distance) for a constant received power with different values of α

$\alpha = 2.8$

$\alpha = 3.0$

$\alpha = 3.2$

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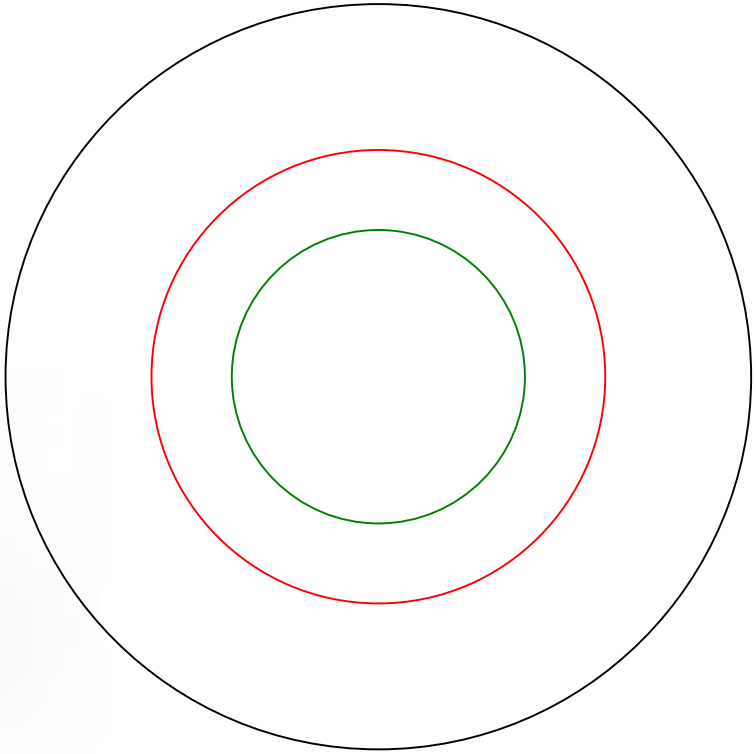
Effect of shadowing spread σ^2

Effect of shadowing uncertainty. For shadowing spread of $\sigma^2 = 4$ dB instead of red circles, the range is between green and black circles with 90% probability



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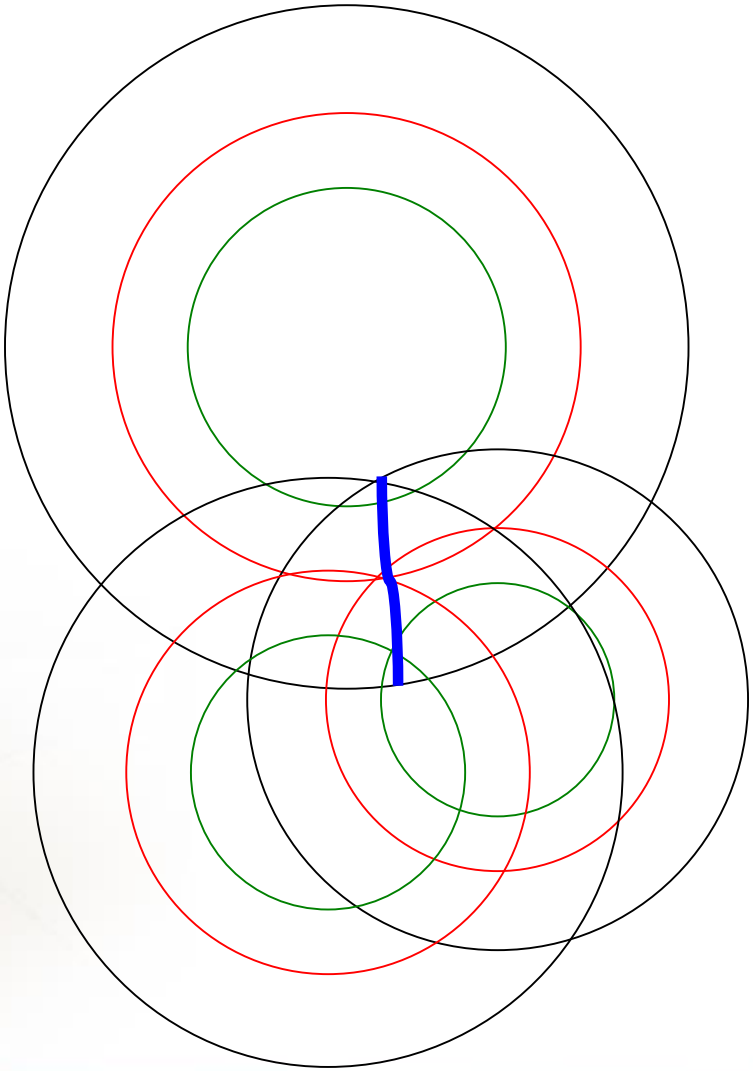
Effect of shadowing uncertainty on trilateration.

Instead of having a single point at the intersection of 3 red circles, we now have a region bounded by green and black circles.



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What is interpolation?

- **Interpolation** is to approximate the value for a given point in some space, when the values at other locations are known.
- **Local Interpolation:** Only the data which fall within the given point's local neighborhood are used for calculating the interpolation values.
- + easy
- - can at best be locally optimum
- **Global Interpolation:** Weighted sum of all data are used to do the interpolation.
- - complex, requires info from all sensors
- + globally optimum solutions are possible

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Interpolation Techniques Considered

Local Techniques

- Nearest Neighbor (used to generate an interpolated map grid in 802.11 networks)
- Natural Neighbor
- Delaunay triangulation based

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Nearest Neighbor Interpolation

- The simplest interpolation technique
- Algorithm: The RSS value at the given node is assigned to all the points inside the Voronoi cell. (All the points inside a cell take the value of that Voronoi site.)
- So the question is: How do we divide the given area to cells?

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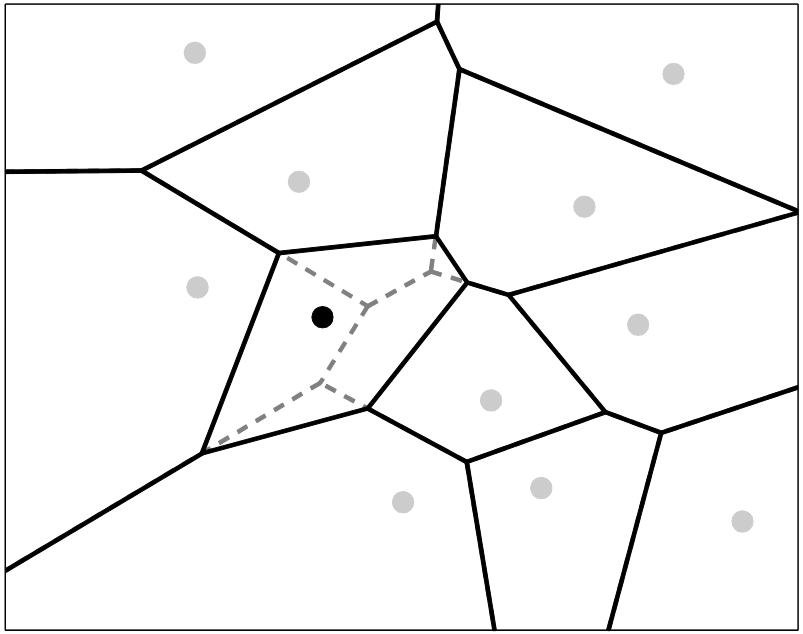
Voronoi Decomposition
(Voronoi Diagram or Dirichlet Tesellation)

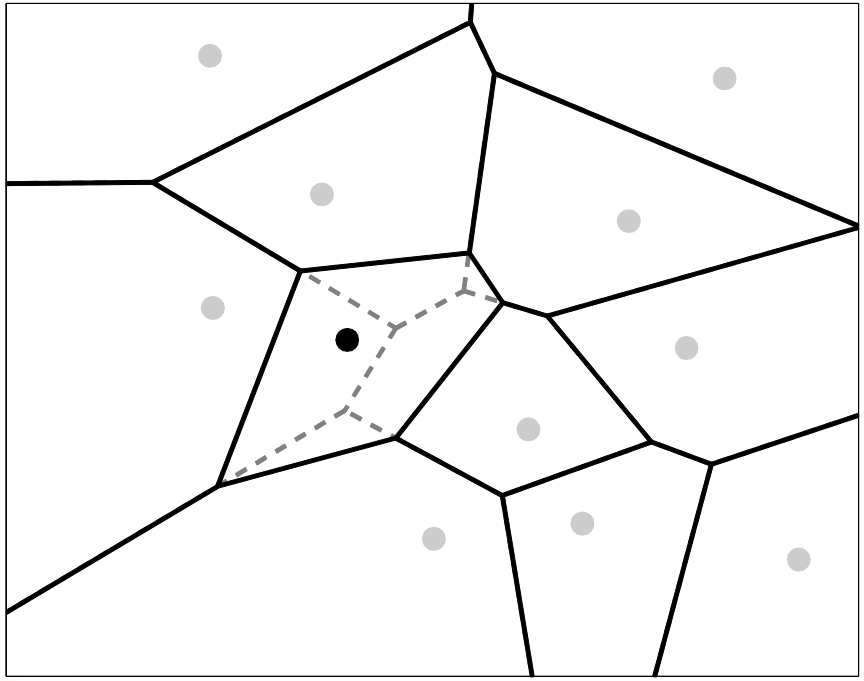
- Definition:
  partitioning of a plane with points into convex polygons such that each polygon contains exactly one generating point and every point in a given polygon is closer to its generating point than to any other.
- ***Tesellation:*** *cover by repeated use of a single shape, without gaps or overlapping.*



Natural Neighbor Interpolation

- Based on Voronoi tesselation of a set of given points. Voronoi tesselation is based on the distances between the given points.
- Given a set of points called Voronoi sites, the decomposition generates a cell for each site such that any point in a given cell is closer to its site than to any other site.

Gray circles : measurement data location
Dark circles : location of the datum to be estimated.
Dashed lines: Tesellation before adding new data
Solid lines : Tesellation after adding new data



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Natural neighbour interpolation consists of the following steps:

1. Assume that the data (input points) are already tessellated. This tessellation serves as a reference for the interpolation. (Voronoi tessellation is the dual representation of Delaunay triangulation.)
2. Include the output point (new point) in the input data set and retessellate the resulting set. The new point adds a new Voronoi cell which overlaps with the cells of the reference.
3. Calculate the interpolated output value at any point x by using a weighted sum of the values at its neighbors x_i for i=1, 2,…M. Where M is the number of neighbors of x.

$$f(\mathbf{x}) = \sum_{i=1}^{M} w_i f(\mathbf{x}_i)$$

The weights w_i are given by the ratio of the area of overlap to the total area of the new cell.

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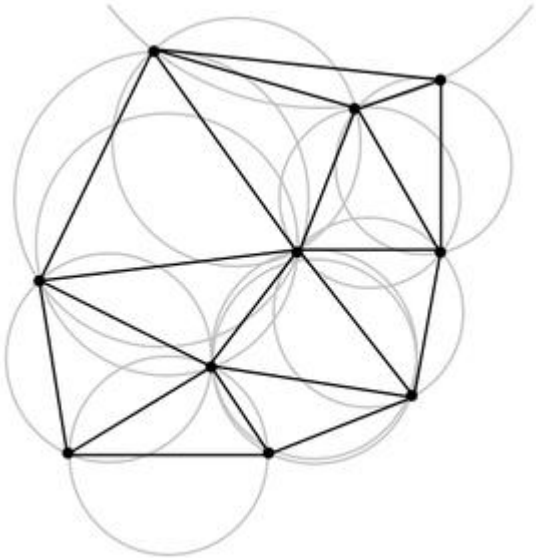
Delaunay Triangulation

- Let P be a set of nodes in the plane, and C, the convex hull of the nodes.
- Draw straight lines (not crossing each other) from nodes on the interior to nodes on the boundary of the convex hull (or to each other), until the entire convex hull is divided into a set of polygons, with all the vertices being elements of P.
- Then, if any of the polygons are not triangles, divide them into triangles by drawing more lines between vertices of the polygons. This will give a triangulation of the set of nodes.



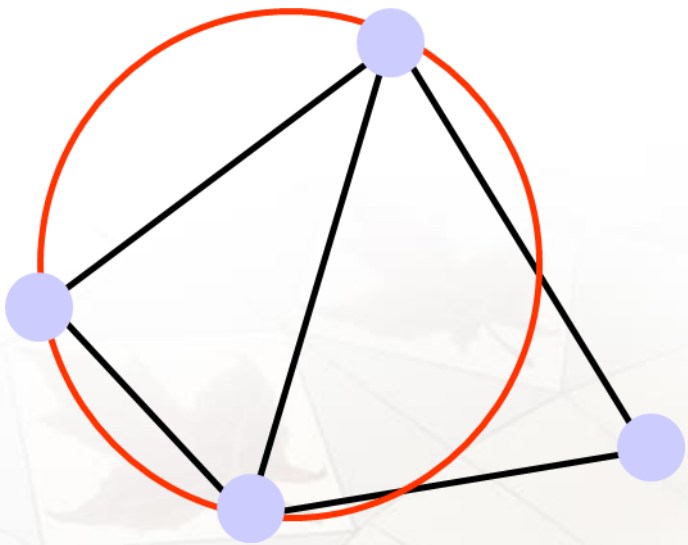
Delaunay Triangulation Properties

- The circumcircle of every triangle contains no other nodes.
- Every line is also contained within some circle which contains no other nodes.
- This triangulation maximizes the minimum angle of all the angles of the triangles in the triangulation.
- This triangulation is unique … except for when 4 or more nodes are on the same circle. This can be avoided by choosing in such a way that the other nodes are outside of the circle. Thus it is assumed that this case does not occur.

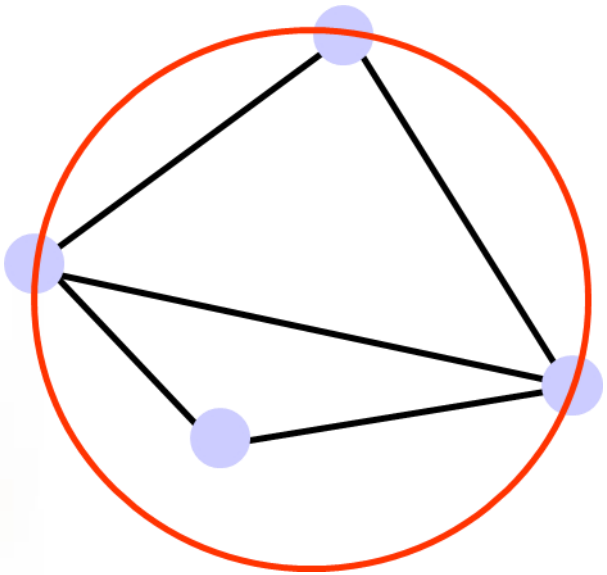
Delaunay Triangulation

Maximizes the minimum interior angle of triangles
No point lies within the circumcircle of a triangle

Yes



No



Polynomial Interpolations

- When the triangulation is obtained, each node in the data set is connected to several others by triangle vertices.
- The interpolated value of any point within the triangle is

$$f(x,y) = \sum_{i=1}^{3} \phi_i(x,y) f_i$$

$\phi_i(\mathbf{x})$ is the interpolating basis function.

triangle's nodes f_i, $i = 1, 2, 3$,

- For the linear interpolation case the basis function is a simple first-order polynomial. $f(x,y) = c_1 x + c_2 y + c_3$.
- Generalization of this process to quadratic and cubic interpolations is straight-forward. Higher order polynomials require larger number of inputs.

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Simulations

Simulation set-up

- Sensors randomly deployed at locations on a planar square of length 1km.
- One primary emitter placed randomly within the square region.
- Log distance path loss model with log-normal shadow fading (independent and correlated) with a path loss exponent of 3.
- 10,000 Monte Carlo simulations with varying number of sensors and shadowing spread values

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Simulations

Performance criteria

RMSE of primary emitter localization error:

rms value of the difference between estimated and actual transmitter location.

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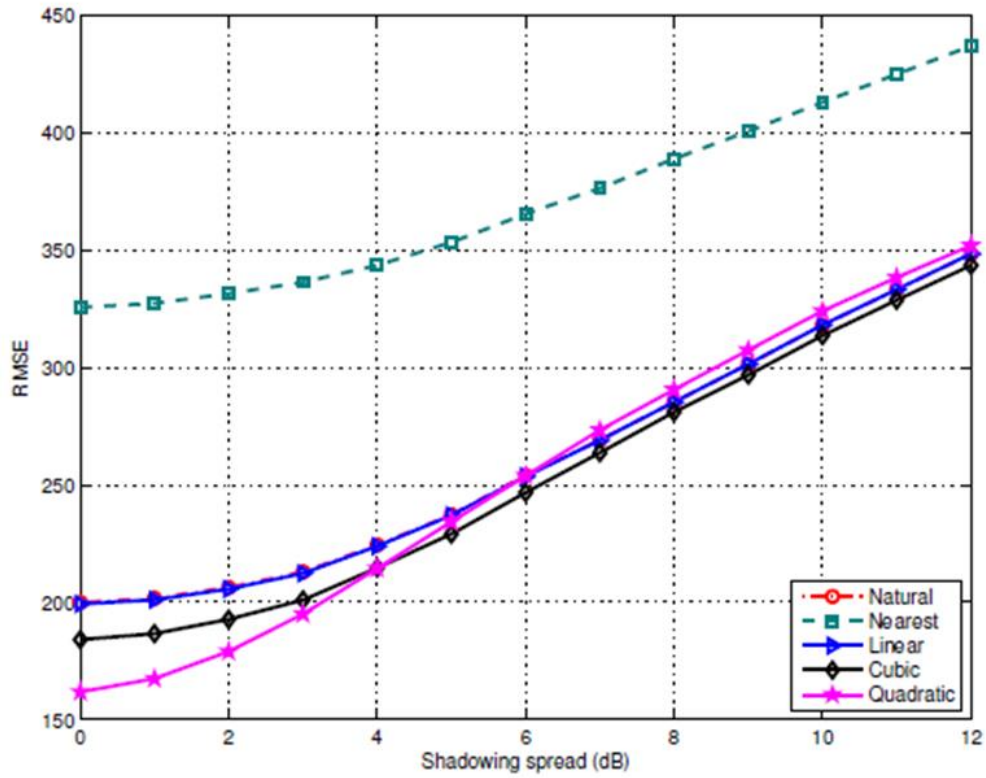
Results for Local Techniques

RMSE performance versus shadowing spread



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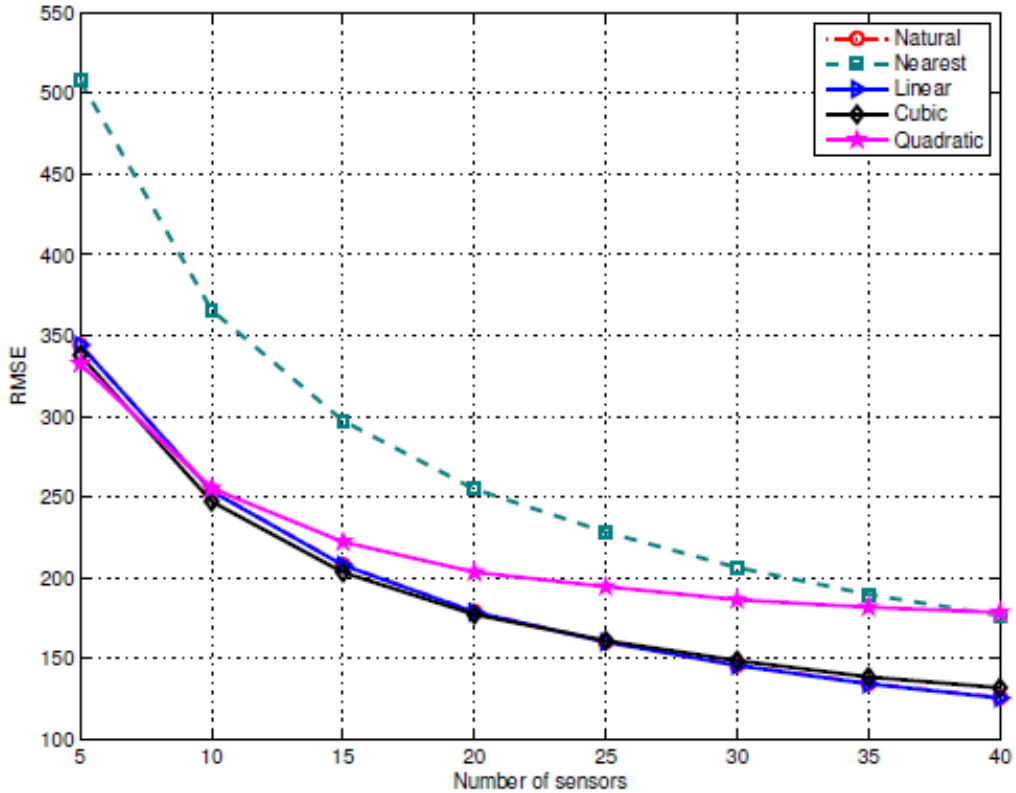
Results for Local Techniques (2)

RMSE performance versus number of sensors



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Back to Estimation Problem

There are several possible approaches, including

1. Least squares solution
2. Steepest descent gradient algorithm
3. EM-like (Expectation maximization) techniques
4. Grid search methods
5. Metropolis Hasting technique

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Least Squares approach to Localization

Let the unknown tx location be $s = (x, y)$.
Denote the known locations of the rx as (x_i, y_i) for $i = 1, 2, \ldots$
The distance between tx and rx i is:

$$d_i^2 = (x - x_i)^2 + (y - y_i)^2$$

Choose $x_1 = 0, y_1 = 0$, for $i > 1$

$$d_i^2 - d_1^2 = x_i^2 + y_i^2 - 2xx_i - 2yy_i$$

$$2xx_i + 2yy_i = x_i^2 + y_i^2 - d_i^2 + d_1^2$$

$$\begin{bmatrix} 2x_2 & 2y_2 \\ \vdots & \vdots \\ 2x_N & 2y_N \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_2^2 + y_2^2 - d_2^2 + d_1^2 \\ \vdots \\ x_N^2 + y_N^2 - d_N^2 + d_1^2 \end{bmatrix}$$

Let $\mathbf{s_e}$ represent the estimate of $\mathbf{s}$

$$\mathbf{H} \cdot \mathbf{s_e} = \mathbf{b}$$

$$\mathbf{s_e} = (\mathbf{H^T H})^{-1} \mathbf{H^T b}$$

Limitations of LS and Steepest Descent Techniques

- LS approach is fine for single tx with known power, having several sensors, with α and σ uncertainties.
- It needs to be modified for unknown power case.
- It does not work for multiple transmitters.

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ML Estimator based on grid search

- Suppose we have a certain area (e.g. 10000 by 10000 m), divided into 100 tiles of 1000 by 1000 m.
- Let N sensors be arbitrarily placed over the area. Each sensor measures a received power.
- Suppose the signal received by each sensor experiences independent log-normal shadowing.
- The ML estimator based on grid search calculates the likelihood function by placing the emitter at the center of each tile. Hence the expected power received (for the distance from the center of tile versus to each sensor location) versus the measured received power are entered into the likelihood function.
- The position that gives the smallest difference between the total measured and assumed received power is the ML estimate.

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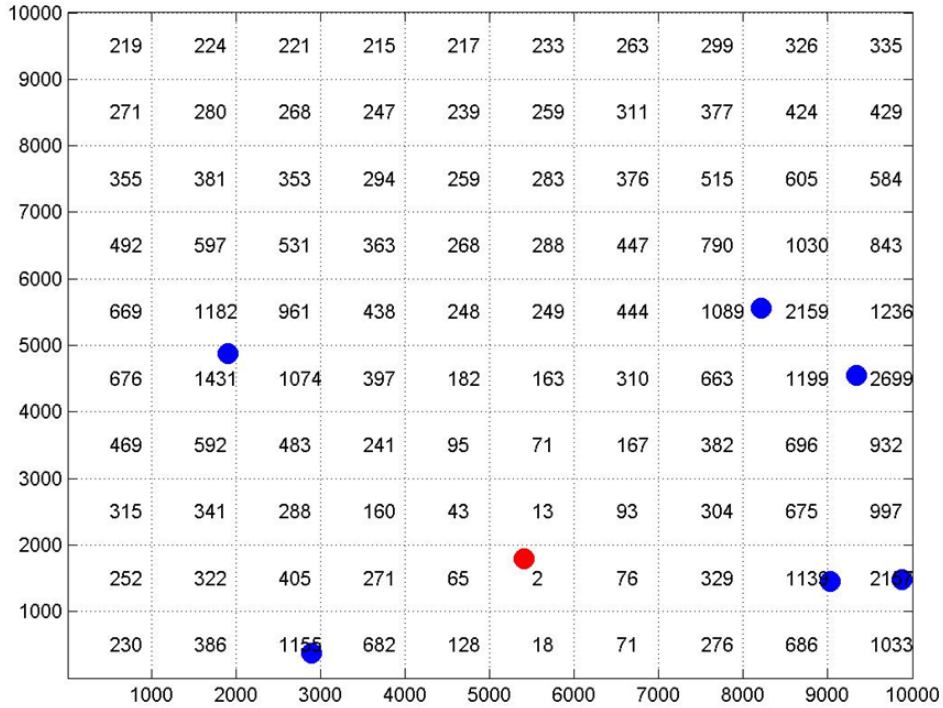
Grid Search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355 381 353 294 259 283 376 515 605 584
492 597 531 363 268 288 447 790 1030 843
669 1182 961 438 248 249 444 1089 2159 1236
676 1431 1074 397 182 163 310 663 1199 2699
469 592 483 241 95 71 167 382 696 932
315 341 288 160 43 13 93 304 675 997
252 322 405 271 65 2 76 329
230 386 682 128 18 71 276 686 1033
1000 2000 3000 4000 5000 6000 7000 8000 9000 10000

Limitations of Grid Search

- The grid search works well for determining the location of a single emitter with known path loss exponent and under the shadowing conditions. (see our ICC 2012 paper)
- It probably would also perform well with path loss exponent uncertainty. (Not done).
- Applying it for

  (a) Multi-emitter case,

  (b) Power unknown case

not done.

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A more general problem formulation

In addition to the unknown tx location as we bring in more parameters into the picture such as

- unknown tx power,
- unknown path loss exponent,
- shadowing uncertainty
- multiple transmitters,

we need to start defining the problem more mathematically.

Pdf with log-normal shadowing

Let P be the txd power and
r_i represent the power received at the i-th sensor at a distance d_i
The mean value of $10\log_{10} r_i$ is denoted μ_i.
Then using the log-distance model as before, the mean rxd power is

$$\mu_i = 10\log_{10}\frac{\beta_0 P}{d_i^{\alpha}}$$

Now let this signal experience log-normal shadowing

$$r_i = \mu_i 10^{\frac{W}{10}}$$

$W \sim N(0, \sigma^2)$ is the random variation due to log-normal shadowing.
The pdf of r_i is

$$p(r_i; \mu_i, \sigma) = \frac{1}{\sigma\sqrt{2\pi}\epsilon r_i} e^{-\frac{(10\log_{10} r_i - \mu_i)^2}{2\sigma^2}}$$

where $\epsilon = \frac{\ln 10}{10}$

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Parameter Estimation

Let the emitter be located at $\mathbf{E} = [E_x E_y]$
Let θ represent the vector of unknown parameters, $\theta = [P\mathbf{E}]$
Assuming iid r_i from N receivers $\mathbf{r} = [r_1\ r_2 \ldots r_N]$
the joint pdf can be written as

$$p(\mathbf{r}|M,\theta) = \prod_{i=1}^{N} \frac{1}{\sigma\sqrt{2\pi}\epsilon r_i} e^{-\frac{(10\log_{10} r_i - \mu_i)^2}{2\sigma^2}}$$

where M represents signal model (e.g. α and noise statistics)
Using the Bayes rule, the joint posterior of the unknown parameters becomes:

$$p(\theta|M,\mathbf{r}) \propto p(\mathbf{r}|M,\theta)p(\theta|M)$$

Obtaining marginal posteriors of location and power parameters require integration over the unknown parameters which cannot be done analytically for our case.

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Generating samples from the posterior density

- A numerical solution to this problem can be obtained by generating samples from the posterior density and then using these samples to make inferences about the unknown parameter.
- One possible approach to solve this problem numerically is to use Monte Carlo Markov Chain (MCMC) methods.

Metropolis-Hastings

- Monte Carlo Markov Chain (MCMC) methods are a class of iterative algorithms for sampling probability distributions based on constructing a Markov chain that has the desired distribution as its equilibrium distribution.
- Metropolis-Hastings (MH) is a MCMC algorithm in which sample generation is based on using a proposal density that permits drawing samples from a target density.

A little digression: generating a pdf

- What are some of the major techniques for generating pdf's?

- Inverse transform technique:
  Samples from f(x) are desired. F(x) (which is the cdf of x) and its inverse are known. Then using uniform distribution, and the inverse transform, samples x are generated.

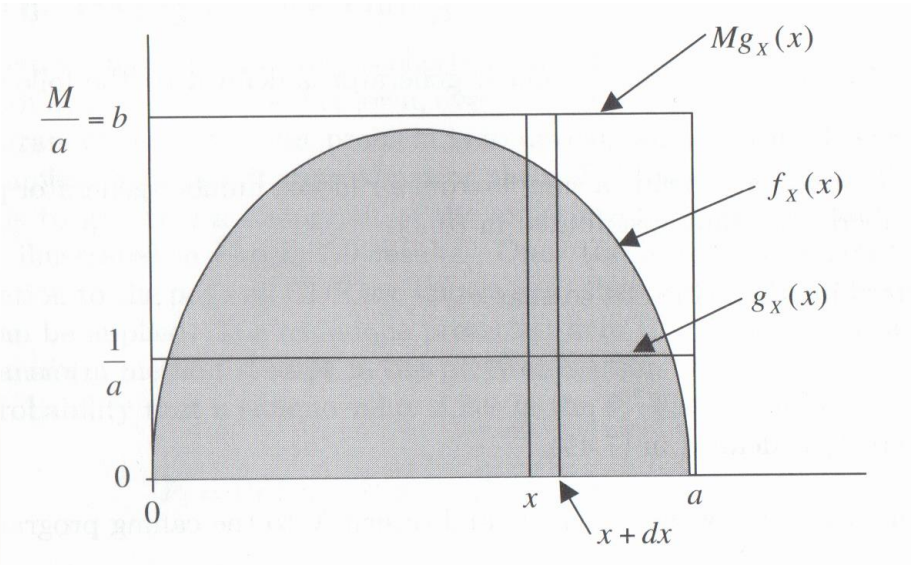
- Accept-Reject Technique (we'll use it later in Metropolis Hasting)

Accept-Reject Technique

- Suppose we want to generate samples from f(x) which we know the functional form up to a constant M.
- We can generate samples from a simpler density g(x).
- Choose a constant M such that Mg(x) ≥ f(x) for all x.

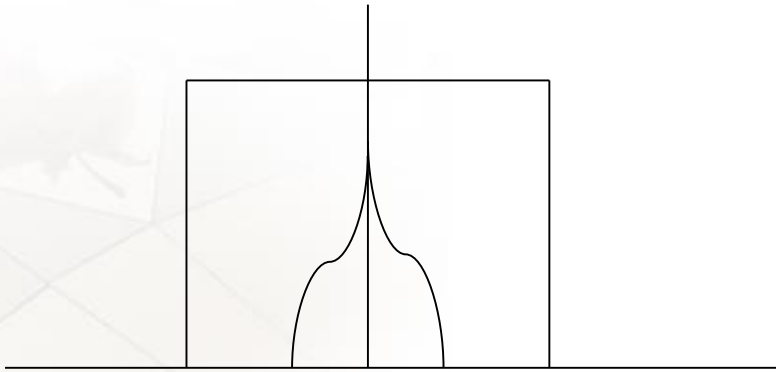
Algorithm

1. Generate Y~g, U ~ U [0 1];
2. If U< f(Y)/Mg(Y), then Accept X=Y
3. Otherwise return to 1

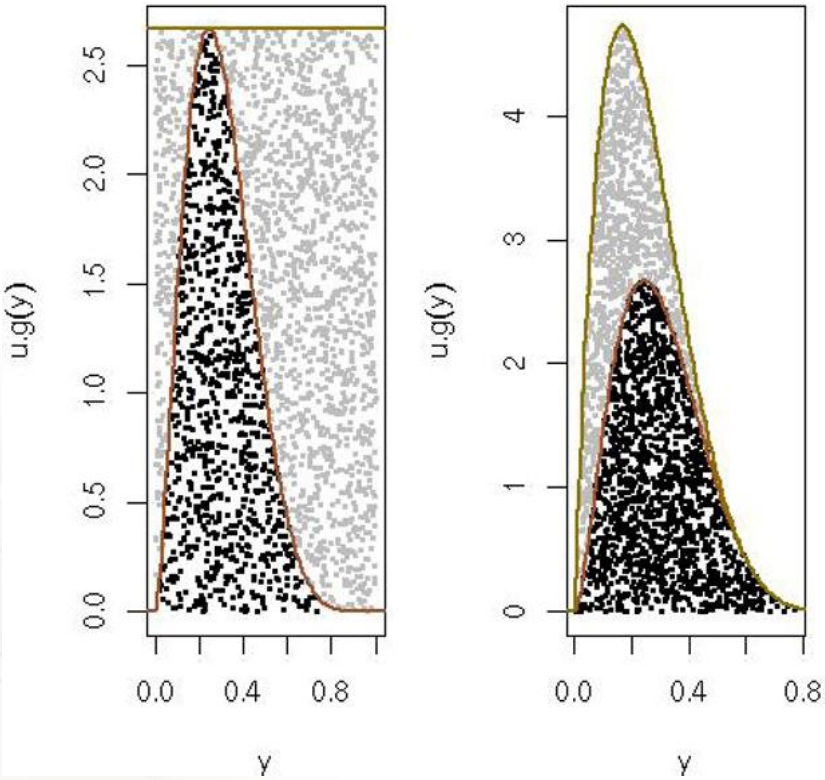


Selecting a good proposed pdf

- Selecting a uniform pdf as proposed (instrumental) density for certain target pdf's may yield extremely low acceptance ratios as can be seen below.



Increase acceptance from 36% to 58%



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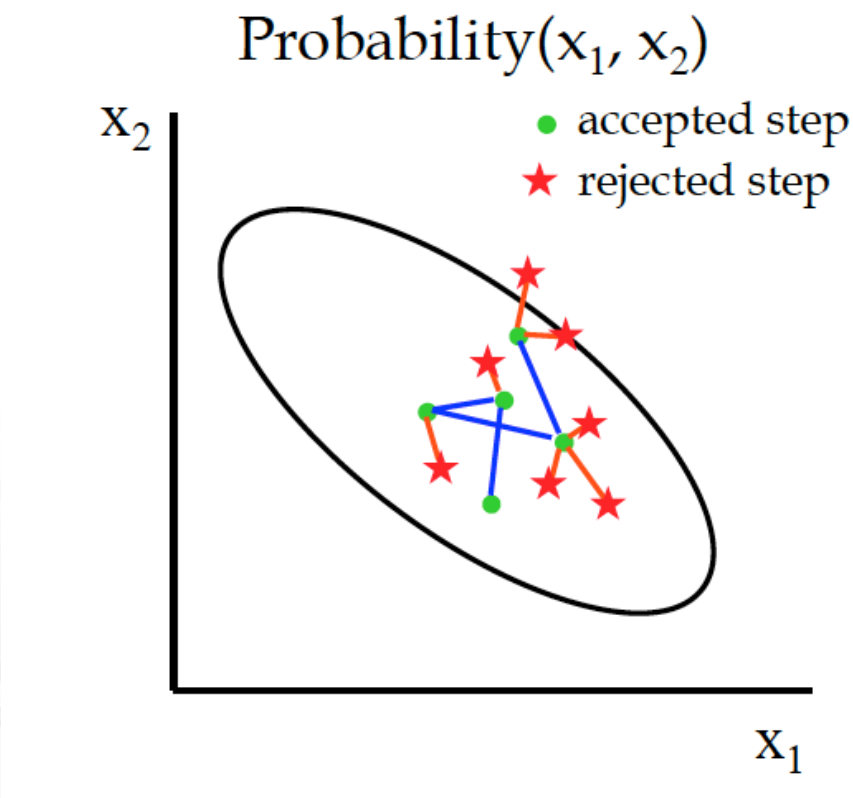
Metropolis-Hastings

MH algorithm starts by assigning random initial values for the parameters. A new value is proposed for a randomly selected parameter using its proposal density. The acceptance probability of the proposed sample is calculated using:

$$\eta(\theta^{(t)}, \theta^*) = \min\{1, \frac{p(\theta^*|M, \mathbf{r})}{p(\theta^{(t)}|M, \mathbf{r})} \frac{q(\theta^{(t)}|\theta^*)}{q(\theta^*|\theta^{(t)})}\}$$

where $q(\cdot)$ is the proposal density and $\theta^{(t)}$ and θ^* are the current and proposed states of the parameters respectively.

Convergence in Metropolis-Hastings



Algorithm 1 M-H algorithm

```
Initialize θ^(t), t ← 0
repeat
    Sample u ~ U(0,1)
    if u ≤ γ then
        Propose a new emitter location S*
        Evaluate η and sample u ~ U(0,1)
        if u ≤ η then
            Accept the proposed position S*
        else
            Keep the old position
        end if
    else
        Propose a new emitter power P*
        Evaluate η and sample u ~ U(0,1)
        if u ≤ η then
            Accept the proposed power P*
        else
            Keep the old power
        end if
    end if
    t ← t + 1
until t = T_max
```

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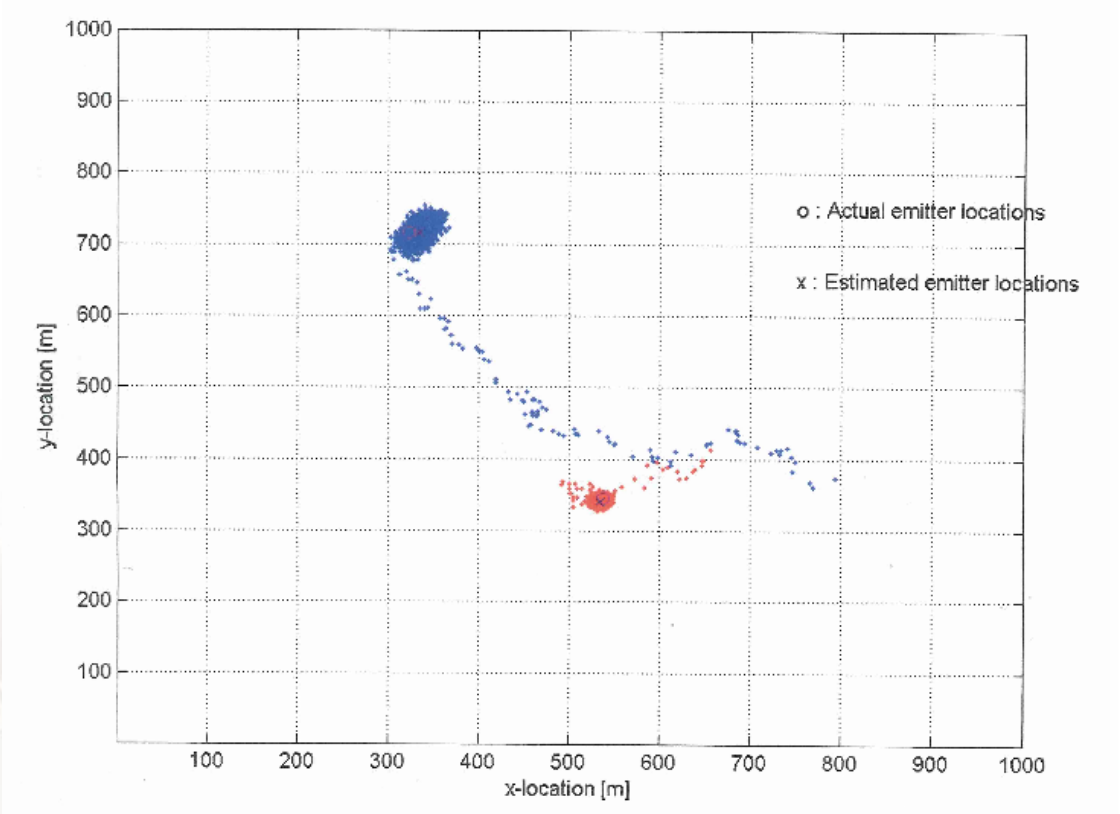
2 individual convergence examples



4 simultaneous convergences example

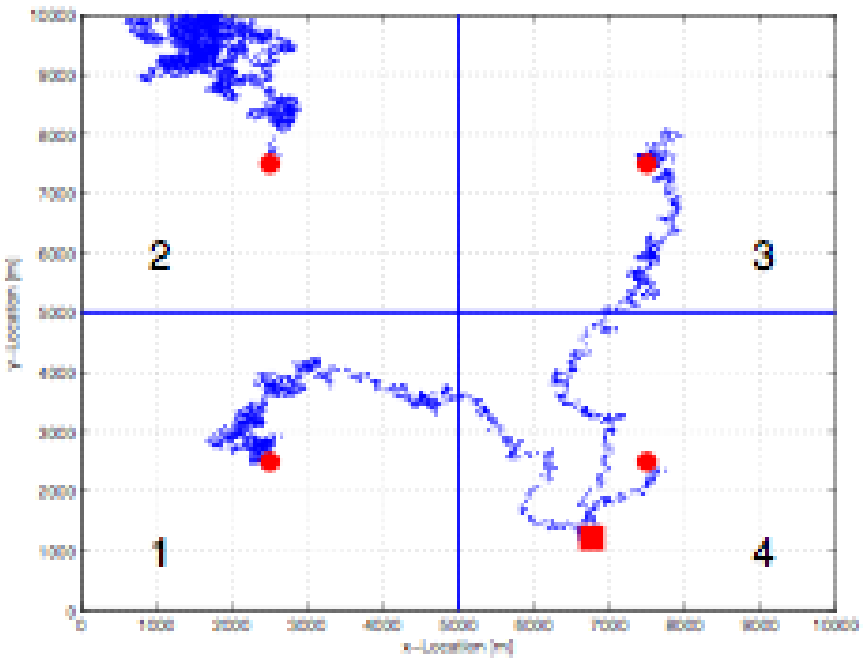


Fig. 1. Traces of four Markov chains started at the quadrant centers.

Outstanding Issue

- We have only considered single emitter with log-normal shadowing
- We need to compare with the literature, such as quasi-EM
- We need to introduce variations in path loss exponent and evaluate the performance
- We need to generalize and adapt for the multi-emitter case
- Very few works that considers directive antennas
- Very few works that considers multipath in localization

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References

[1] N. Miliou, A. Moustakas, and A. Polydoros, "Interference source localization and transmit power estimation under log-normal shadowing", 19th European Signal Processing Conference, Barcelona, Spain, 2011.

[2] M. Zafer, Bong Jun Ko, and I. W.-H. Ho, "Transmit power estimation using spatially diverse measurements under wireless fading", IEEE/ACM Trans. On Networking, vol. 18, no 4,pp. 1171-1180, 2010.

[3] J. K. Nelson, M. U. Hazen and M. Gupta, "Global optimization for multiple transmitter localization".

[4] J. K. Nelson, M. Gupta, J. E. Almodovar and W. H. Mortensen, "A quasi EM method for estimating multiple transmitter locations", IEEE Signal Processing Letters, vol. 16,no 5, pp.354-357, 2009.

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References (ctd)

[5] A. O. Nasif and B. L. Mark, “Measurement clustering criteria for localization of multiple transmitters”.

[6] S. Ureten, A.Yongaçoğlu, E. Petriu, “Interference Map Generation Based on Delaunay Triangulation in Cognitive Radio Networks”, 13th IEEE Conf. on Signal Proc.Applications in Wireless Comm. (SPAWC-2012) June 2012.

[7] S. Ureten, A.Yongaçoğlu, E. Petriu, “A Comparison of Interference Cartography Generation Techniques in Cognitive Radio Networks”, IEEE Int’l Conference on Comm. (ICC-2012), Ottawa, June 2012.

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